**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: B**

servicing transmissions is normally distributed

*μ* = 45 minutes

*σ* = 8 minutes.

*Probability of Service manage meets deadline is stats.norm.cdf (50,45,8)=0.734014470*

probability that the service manager cannot meet his commitment= 1- *stats.norm.cdf (50,45,8)*

*=1-0.734014470=0.26598553*

*Note: Jupyter Notebook file attached*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

1. ANS: False

mean *μ* = 38

Standard deviation *σ* =6

Around 68% of the data falls within one standard deviation of the mean

P (38-1*σ<=X<=38+1σ ) equal to 68.27%*

P (38-6*<=X<=38+6 ) equal to 68.27%*

P(32*<=X<=44) equal to 68.27%*

1. ANS: True

Z=(X-µ)/ *σ*

Z= (30-38)/6=-8/6= -1.3333

P(X≤30)=p(Z≤-1.3333)= 0.09176(using Z table)

Expected count=0.09176\*400= 36.704

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: 2 *X*1  will be greater than *X*1 + *X*2 . SINCE  *X*1 and *X*2 are *iid* normal random variables (independent, identically distributed random variables) , their sum is approximately normal. If X1 and X2 are normally distributed then the sum of the random sample will be exactly same.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: D**

μ = 100, σ =20

stats.norm.ppf (0.995,100,20)= 151.516586070978

stats.norm.ppf (0.005,100,20)= 48.483413929021985

*Note: Jupyter Notebook file attached*

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

Profit1

μ =5, σ =3

Profit2

μ =7, σ =4

A

stats.norm.ppf (0.025,45\*5,3) = 219.1201

stats.norm.ppf (0.975,45\*5,3) = 230.8798

stats.norm.ppf (0.025,45\*7,4) = 307.1601

stats.norm.ppf (0.975,45\*7,4) = 322.8398

The Rupee Range will be

[219.1201, 230.8798] + [307.1601, 322.8398] = [526.2802,553.7196]

B

stats.norm.ppf (0.05,45\*5,3) = 220.0654391191456

stats.norm.ppf (0.05,45\*7,4) = 308.4205854921941

5th percentile of profit (in Rupees) = 220.0654+ 308.4205= 528.4859

C.

1st Division

**No***te: Jupyter Notebook file attached*